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PARTICULAR STREAM SURFACES IN CONICAL GAS FLOWS

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Considering the field of conical flow of an ideal perfect gas near conical stream surfaces, we show that ordinary (regular) stream surfaces which are constant-entropy surfaces (isentropes) can coexist with particular stream surfaces characterized by distributed variable entropy. These particular surfaces are envelopes of the field-of-flow isentropes and can be contiguous with the regular stream surfaces without disrupting the continuity of the stream surface either in the vicinity of the particular stream surface or in the vicinity where the two surfaces meet. The results obtained enable us to postulate a pattern of nonsymmetric flow past conical bodies with a continuous and unique distribution of gasdynamic parameters in the field of flow, and to infer that this pattern is free of singular points [1].

1. Let us consider the flow of an ideal perfect gas conically symmetric with its center at the point O ; we assume that the field of flow contains a conical stream surface S on which the normal component of the flow velocity is equal to zero by definition. The stream surface S is represented by the curve S on the sphere of unit radius with its center at O (Fig. 1). We assume that in the curvilinear coordinate system η, ζ the stream surface S corresponds to $\eta = 0$ and the lines $\zeta = \text{const}$ correspond to the normals to S. In such coordinates the equations of gas motion are, for example [2], of the form

$$\begin{gathered} wu_\zeta + Xvu_\eta - X(w^2 + v^2) = 0 \\ wv_\zeta + Xvu_\eta + Xuv + Yw^2 = -\rho^{-1} Xp_\eta \\ ww_\zeta + Xvw_\eta + X_{uw} - Yvw = -\rho^{-1} p_\zeta \\ w\rho_\zeta + vX\rho_\eta + \rho(w_\zeta + Xv_\eta + 2Xu - Yv) = 0 \\ 2\varkappa(\varkappa - 1)^{-1} p + \rho(u^2 + v^2 + w^2) = \rho V^2_{\max} \end{gathered} \qquad (1.1)$$

where u, v, w are the components of the velocity vector along the radius r and along the coordinates η, ζ, respectively; $V_{\max}$ is constant throughout the stream; p, ρ, $\varkappa$ are the pressure, density, and ratio of specific heats of the gas, respectively, and $k(\zeta)$ is the curvature of S. Equations (1.1) also contain the symbols

$$X = \cos\eta - k\sin\eta, \qquad Y = \sin\eta + k\cos\eta \tag{1.2}$$

We begin by considering stream surfaces S_R near which all of the flow parameters are continuous, have continuous derivatives of arbitrary order with respect to η, and can be expressed as expansions in integer powers of η which converge in some neighborhood of S_R. We shall refer to such stream surfaces S_R as "regular". Stream surfaces S_O not satisfying this condition will be called "particular".

As we shall see below, the stream surface S can consist partly of S_R and partly of S_O.

Expressing the flow parameters as series in η near S_R and recalling that $v = 0$ on S_R, we have

$$u = u_0(\zeta) + \eta u_1(\zeta) + \ldots, \qquad p = p_0(\zeta) + \eta p_1(\zeta) + \ldots \tag{1.3}$$
$$v = \eta v_1(\zeta) + \eta^2 u_2(\zeta) + \ldots, \qquad w = w_0(\zeta) + \eta w_1(\zeta) + \ldots$$
$$\rho = \rho_0(\zeta) + \eta\rho_1(\zeta) + \ldots$$

Substituting these expansions into (1.1), we obtain recursion relations for the series coefficients. The first equation of (1.1) indicates that one of two cases is possible: either $w_0 = u_0'$ or $w_0 = 0$. In the first case the principal terms of the third and fifth equations coincide if the entropy function $\sigma = p\rho^{-\varkappa}$ is constant on S (the case of variable entropy must be rejected because of the incompatibility of the principal terms of these equations). The recursion relations for the series coefficients therefore contain the function $u_0(\zeta)$, which can be defined in arbitrary fashion,

$$u = u_0(\zeta) + O(\eta), \quad v = -\eta\left(2u_0 + u_0'' + u_0'\frac{\rho_0}{\rho_0}\right) + O(\eta^2) \tag{1.4}$$
$$w = u_0' + O(\eta), \quad p = \frac{\varkappa - 1}{2\varkappa}\rho_0(V_{\max}^2 - u_0^2 - u_0'^2) - \eta k\rho_0 u_0'^2 + O(\eta^2)$$

Thus, the first case corresponds to a conical stream surface S_{R1} which coincides with the equal-entropy surface (isentrope); the isentropes of the field of flow near S_{R1} are parallel to the latter (Fig, 2a).

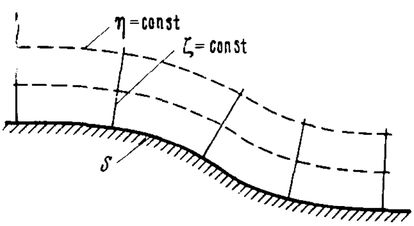

Fig. 1

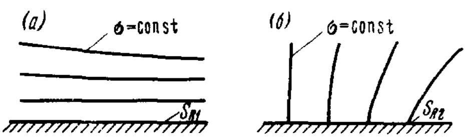

Fig. 2

The second case $w_0 = 0$ is interesting in that it constitutes an example of a conical stream surface S_{R2} on which the entropy can vary. In this case the relations for the series coefficients contain the single arbitrary function $u_0(\zeta)$ and are of the form

$$u_1 = 0,\ u_2 = -u_0 - {}^1/_8 u_0'^2 V_{\max}^2 u_0^{-1} (V_{\max}^2 - u_0^2)^{-1}, \ldots, v_1 = -2u_0, \ldots,$$
$$w_1 = 0,\ w_2 = -{}^2/_3 u_0' V_{\max}^2 (V_{\max}^2 - u_0^2)^{-1}, \ldots, p_0 = \text{const} = p_{00},\ p_1 = 0$$
$$p_2 = -\rho_0 u_0^2, \ldots, \qquad \rho_0 = 2\varkappa p_{00} (\varkappa - 1)^{-1} (V_{\max}^2 - u_0^2)^{-1}, \ldots \qquad (1.5)$$

In the coordinate system η, ζ the slope of the equal-entropy curves is given by

$$d\eta/d\zeta = vX/w \qquad (1.6)$$

From this we see that $d\eta/d\zeta \to \infty$ as $\eta \to 0$, i.e. the isentropes approach the surface S_{R2} along the normals (Fig. 2b), and the pressure at this surface is constant. The case $u_0 = \text{const}$ corresponds to flow with constant entropy.

We note that the surfaces S_{R1} and S_{R2} cannot adjoin each other continuously. This is because of the diametrically oposite directions of the isentropes, which require the existence either of corner points or of isolated singular points in the field of flow.

Stream surfaces (isentropes) are widely used for constructing the field of flow past conical bodies. For example, the authors of studies on nonsymmetric flow past cones [1–4] and delta wings [5, 6] assume that the entropy is constant everywhere on the body surface except at certain isolated points of entropy discontinuity [1].

2. Now let us investigate the possibility of existence of particular conical stream surfaces S_O. We can express the flow parameters in the neighborhood of S_O in the form of the following expansions which ensure the continuity of the flow parameters as S_O is approached, although their first (or higher) normal derivatives may go to infinity:

$$u = u_0(\zeta) + \eta u_1(\zeta) + \ldots + \eta^m [u_1^\circ(\zeta) + \eta u_2^\circ(\zeta) + \ldots] \qquad (2.1)$$
$$v = \eta v_1(\zeta) + \eta^2 v_2(\zeta) + \ldots + \eta^m [v_1^\circ(\zeta) + \eta v_2^\circ(\zeta) + \ldots]$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$\rho = \rho_0(\zeta) + \eta\rho_1(\zeta) + \ldots + \eta^m [\rho_1^\circ(\zeta) + \eta\rho_2^\circ(\zeta) + \ldots]$$
$$(0 < m < 1)$$

Substituting (2.1) into (1.1), we find that it is possible to construct series for $m = {}^1/_2$ with noncontradictory recursion relations. To simplify the derivation of these relations, instead of applying (2.1), we first set $\tau = \sqrt{\eta}$ in Eqs. (1.1) and then substitute into them the expansions of the functions in series in integer powers of τ.

$$u = u_0^*(\zeta) + \tau u_1^*(\zeta) + \ldots, \quad v = \tau v_1^*(\zeta) + \tau^2 v_2^*(\zeta) + \ldots \qquad (2.2)$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$\rho = \rho_0^*(\zeta) + \tau\rho_1^*(\zeta) + \ldots$$

It is clear that the two above procedures are equivalent for $m = {}^1/_2$.

The second and fourth equations of system (1.1) require that $v_1^* = 0$, $p_1^* = 0$; the first equation points to the existence of two cases: either $w_0^* = u_0^{*\prime}$ or $w_0^* = 0$. The first case corresponds to a constant entropy on S_{O1} (this follows from the third and fifth equations), and the coefficients of the expansion are given by recursion relations containing the single arbitrary function $u_0^*(\zeta)$. Thus, the surface S_{O1} is an isentrope, and the character flow in the vicinity of this surface is qualitatively the same as in the case of S_{R1}.

We note that noncontradictory expansions with other fractional values of the exponent m can exist in the neighborhood of S_{O1}.

The most interesting stream surfaces are S_{O_2} which correspond to the second case $w_0{}^* = 0$, since the entropy on these surfaces turns out to be variable. The coefficients of the expansions generally contain the two arbitrary functions $u_0{}^*(\zeta)$ and $u_1{}^*(\zeta)$, i.e. they are somewhat more arbitrary than the expansions near the isentropes, and the expansions are noncontradictory for $m = {}^1/_2$ only,

$$u = u_0{}^*(\zeta) + \eta^{1/2}u_1{}^* + O(\eta), \qquad v = -2\eta u_0{}^*(\zeta) + O(\eta^{3/2})$$
$$w = \eta^{1/2}u_0{}^*u_1{}^* / u_0{}^{*\prime} + O(\eta), \quad \rho = 2\varkappa\, p_{00}\, (\varkappa - 1)^{-1}\, (V_{\max}{}^2 - u_0{}^2)^{-1} + \ldots$$
$$p = p_{00} - \rho_0\, \eta^2\, [u_0{}^{*2} + {}^1/_2 k\, (u_0{}^*u_1{}^* / u_0{}^{*\prime})^2] + \ldots \tag{2.3}$$

The pressure p_{00} at the surface S_{O2} is constant, while the density ρ, and therefore the entropy, are variable. The velocity w of transverse flow vanishes on S_{O2}; the velocity u tends to finite values; the normal derivatives of the velocities u, w and of the density ρ go to infinity as $\eta^{-1/2}$ with diminishing distance from the stream surface, although the pressure gradient and its derivative are continuous. Let us consider the pattern of isentropes in the vicinity of S_{O2}. Substituting the principal terms of (2.3) into Eq. (1.6), we obtain the following equation $\eta_i = \eta_i(\zeta, \sigma)$ for the isentropes near S_{O_2}:

$$\eta_i \approx (C_i - \zeta\, u_0{}^{*\prime} / u_1{}^*)^2 + O(\eta^{3/2}) \tag{2.4}$$

where C_i is constant along each isentrope.

Overlooking the degenerate case $u_0 = \text{const}$, we can now readily verify that from every point ζ_0 belonging to S_{O2} there emerges a single isentrope with the corresponding entropy value (Fig. 3). The isentropes near S_{O2} are parabolas tangent to S_{O2} at the points ζ_0.

$$\eta_i \approx (u_0{}^{*\prime} / u_1{}^*)^2 \cdot (\zeta - \zeta_0)^2 \tag{2.5}$$

The surface S_{O2} is thus the envelope of the family of isentropes.

The fact that the velocity components v and w vanish at the stream surface S_{O2} also imply that the streamlines of the field of flow asymptotically approach the radial generatrices of the stream surface S_{O2}.

Thus, a conical flow can contain conical stream surfaces over which the entropy varies. Moreover, the series expansions of the flow parameters which permit departure from these surfaces are highly arbitrary (they contain two arbitrary functions).

This fact suggests that such special stream surfaces are in no way exceptional, and can occur in flows past conical bodies of varying configuration. It is, of course, generally impossible for the entire surface of a streamlined body to be contiguous with the particular stream surface S_{O_2} exclusively (although exceptions to this general rule appear to be possible). It is therefore important to investigate the possibility of continuous contiguity of a regular and a particular stream surface.

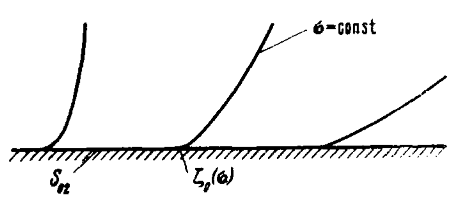

Fig. 3

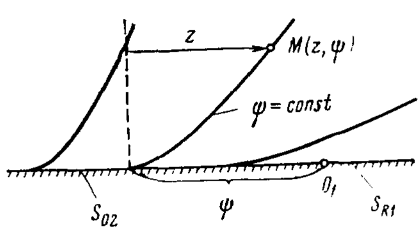

Fig. 4

3. Let us consider a portion of a sufficiently smooth conical stream surface S and suppose that at the point O_1 with the coordinate $\zeta = \zeta_1$ the particular and regular stream surfaces come in contact; the surface S_{O_2} lies to the left of O_1 and S_{R_1} to its right (Fig. 4). We shall show that the flow pattern in the vicinity of the point O_1 can be as follows: isentropes emerge from S_{O_2}; the curvature of these isentropes gradually decreases to zero as the point O_1 is approached from the left; the limiting isentrope emerges from O_1 along the axis $\eta = 0$. Series of the form (2.2) are not suitable for describing the flow in the vicinity of the point O_1. In fact, (2.5) implies that as $\zeta \to \zeta_1$ the quantity $u_0^{*\prime} / u_1^*$ must tend to zero. However, this requires, for example, that the coefficients of the expansion for (2.3) go to infinity. This flow zone requires the use of power expansions in other variables which can be extended to the left and to the right of the point O_1 for matching with series (1.3) and (2.2). To this end we introduce the variables (Fig. 4)

$$z = {}^1/_2 [\zeta - \zeta_1 + \sqrt{(\zeta - \zeta_1)^2 + 4\sqrt{\eta}}]$$
$$\psi = -{}^1/_2 [\zeta - \zeta_1 - \sqrt{(\zeta - \zeta_1)^2 + 4\sqrt{\eta}}] \qquad (3.1)$$

The inverse transition (for $\eta \geqslant 0$) can be effected by means of the formulas

$$\zeta = \zeta_1 + z + \psi, \qquad \eta = z^2\psi^2 \qquad (3.2)$$

The curves $\psi = \text{const}$ are parabolas tangent to the axis $\eta = 0$; the quantity ψ as well as the curvature of the parabolas tend to zero as the points of tangency approach O_1, so that the last parabola $\psi = 0$ coincides with the line $\eta = 0$, $\zeta \geqslant \zeta_1$. The variable z constitutes the projection on the axis $\eta = 0$ of the distance from the point (z, ψ) under consideration to the origin of the corresponding parabola. It turns out that in these variables system of gasdynamic equations (1.1) has solutions which satisfy the no-leak condition at S; these solutions take the form of the following power series:

$$u = u_0^{**}(\psi) + zu_1^{**}(\psi) + \ldots, \quad p = p_{00} + zp_1^{**}(\psi) + \ldots \qquad (3.3)$$

Equations (1.1) rewritten in the variables z, ψ are

$$D(u) - X2z\psi(\psi + z)(w^2 + v^2) = 0 \qquad (3.4)$$
$$D(v) + 2z\psi(\psi + z)(Xuv + Yw^2) = \rho^{-1}X(p_z + p_\psi)$$
$$D(w) + 2z\psi(z + \psi)(Xuw - Yvw) = 2z\psi/\rho^{-1}(zp_z - \psi p_\varphi)$$
$$D(\rho) + 2\rho z\psi(zw_z - \psi w_\psi) + X\rho(v_z + v_\psi) +$$
$$+ 2\rho z\psi(\psi + z)(2Xu - Y_v) = 0$$
$$p\, 2\varkappa(\varkappa - 1)^{-1} + \rho(u^2 + v^2 + w^2) - \rho V^2_{\max} = 0$$
$$D(\cdot) \equiv (2z^2\psi w + Xv)\,\partial(\cdot)/\partial z + (Xv - 2z\psi^2 w)\,\partial(\cdot)/\partial\psi$$

Substituting (3.3) into this system, we find that the fourth equation yields $v_1 = 0$ and the second equation requires $p_1 = p_2 = p_3 = 0$. Under these conditions we obtain noncontradictory recursion relations of the coefficients of series (3.3); as is the case with (2.2), these relations contain the two arbitrary functions $u_0^{**}(\psi)$ and $u_1^{**}(\psi)$. The principal terms of the expansions are of the form

$$u = u_0^{**} + zu_1^{**} + \ldots, \qquad v = -2z^2\psi^2 u_0^{**} + \ldots$$

$$w = -z\frac{u_0^{**}}{(u_0^{**})'}[(u_0^{**})' + u_1^{**}] + \ldots$$

$$p = p_{00} - z^4\rho_0\left\{\psi^4(u_0^{**})^2 + {}^1/_2 k\psi^2\frac{(u_0^{**})^2}{(u_0^{**})'^2}[(u_0^{**})' + u_1^{**}]^2\right\} + \ldots \qquad (3.5)$$

$$\rho = \rho_0 + O(z) = 2p_{00}\varkappa(\varkappa - 1)^{-1}[V_{\max}^2 - (u_0^{**})^2]^{-1} + \ldots$$
$$p_{00} = \text{const}$$

Hence, under certain conditions imposed on the functions $u_0^{**}(\psi)$ and $u_1^{**}(\psi)$ in order to ensure the convergence of the power series the indicated flow pattern in the neighborhood of O_1 is indeed possible; moreover, the solutions to the left of the point O_1 take the form of series in z which can be reconstructed into series in $\sqrt{\eta}$ with principal terms of the form (2.2). In fact, the variables z, ψ for $\zeta < \zeta_1$ and for $\eta \ll (\zeta - \zeta_1)^2$ (near $\eta = 0$, but far away from the point O_1) can be written as

$$z \approx \frac{\sqrt{\eta}}{\zeta_1 - \zeta} + O\left(\frac{\sqrt{\eta}}{(\zeta_1 - \zeta)^3}\right), \qquad \psi \approx (\zeta_1 - \zeta) + O\left(\frac{\sqrt{\eta}}{\zeta_1 - \zeta}\right) \tag{3.6}$$

To the right of the point O_1 near $\eta = 0$ we have

$$z \approx (\zeta - \zeta_1) + O\left(\frac{\sqrt{\eta}}{\zeta - \zeta_1}\right), \qquad \psi \approx \frac{\sqrt{\eta}}{\zeta - \zeta_1} + O\left(\frac{\sqrt{\eta}}{(\zeta - \zeta_1)^3}\right) \tag{3.7}$$

and the principal terms of series (3.5) assume a form corresponding to (1.3).

Thus, the fact of existence of noncontradictory expansions of the solution into power series in the hypothetical flow pattern in the vicinity of joining of the regular and particular stream surfaces points to the possibility of such flows, although a complete proof of the existence of the solution requires the imposition of conditions on the arbitrary functions to ensure the convergence of the solutions employed.

4. If a flow is symmetric relative to some plane $\zeta = 0$, then the derivative $\partial u / \partial\zeta \to 0$ as $\zeta \to 0$ on the particular stream surface S_{O2}. Hence, other expansions are required in the neighborhood of the point $\zeta = 0$ of the particular stream surface S_{O2}, since expansions (2.3) lack meaning there.

We were able in this case to construct the solution in the spherical coordinate system r, θ, φ with the axis $\theta = 0$ passing through $\zeta = 0$ (Fig. 5). As we know, Eqs. (1.1)

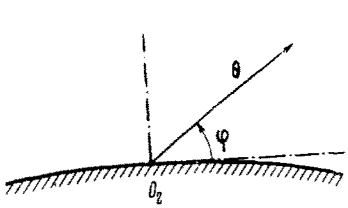

Fig. 5

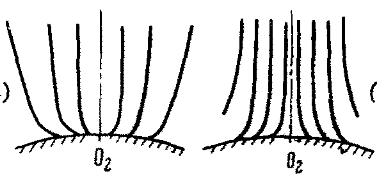

Fig. 6

can be written in this coordinate system as

$$\begin{gathered}
q \sin\theta u_\theta + w u_\varphi - q^2 \sin\theta - w^2 \sin\theta = 0 \\
q \sin\theta q_\theta + w q_\varphi + q u \sin\theta - w^2 \cos\theta = -\rho^{-1} \sin\theta p_\theta \\
q \sin\theta w_\theta + w u_\varphi + u w \sin\theta + q w \cos\theta = -\rho^{-1} p_\varphi \\
q \sin\theta \rho_\theta + w \rho_\varphi + \rho(2u \sin\theta + \sin\theta q_\theta + q \cos\theta + w_\varphi) = 0 \\
2p\varkappa(\varkappa - 1)^{-1} + \rho(u^2 + q^2 + w^2) = \rho V_{\max}^2
\end{gathered} \tag{4.1}$$

where u, q, w are the components of the velocity vector which correspond to the directions of variation of r, θ, φ.

Let the stream surface S be defined by the series

$$\varphi = a_1\theta + a_2\theta^2 + \ldots \quad \text{for } \varphi \leqslant {}^1/_2\pi, \quad \varphi = \pi - a_1\theta - a_2\theta^2 \ldots \quad (\varphi > {}^1/_2\pi) \tag{4.2}$$

With allowance for symmetry and for the no-leak condition at S, we can express the flow parameters in (4.1) as the expansions

$$u = u_0^\circ(\varphi) + \theta u_1^\circ(\varphi) + \ldots, \quad q = \theta q_1^\circ(\varphi) + \theta^2 q_2^\circ(\varphi) + \ldots \tag{4.3}$$

$$w = \theta w_1^\circ(\varphi) + \theta^2 w_2(\varphi) + \ldots, \quad p = p_{00} + \theta p_1^\circ(\varphi) + \ldots$$

$$\rho = \rho_0^\circ(\varphi) + \theta\rho_1^\circ(\varphi) + \ldots$$

Several authors, e. g. those of [7, 8], have investigated conical flows in the neighborhood of the point of symmetry O_2 with the aid of such expansions.

The first equation yields

$$(u_0^\circ)' w_1^\circ = 0 \tag{4.4}$$

The case $(u_0^\circ)' \neq 0$, $w_1^\circ = 0$ requires the existence of a singular point in the entropy and density distributions (called the "Ferri [1] entropy singularity" in several papers). In this case, which is investigated in detail in the aforementioned papers, the isentropes converge to the point O_2, so that the case corresponds to the regular stream surface S_R (or S_{O_1}).

Let us consider the case $(u_0^\circ)' = 0$, $w_1^\circ \neq 0$, which, as already noted, also yields noncontradictory recursion formulas for the coefficients of series (4.2). For the first terms of the expansion we obtain

$$q_1^\circ u_1^\circ + w_1^\circ u_1^{\circ\prime} = 0 \qquad q_1^{\circ 2} + w_1^\circ q_1^{\circ\prime} + u_0^\circ q_1^\circ = -\frac{2}{\rho_0^\circ} p_2^\circ \tag{4.5}$$

$$q_1^\circ w_1^\circ + w_1^\circ w_1^\circ + u_0^\circ w_1^\circ = -p_2^{0\prime}/\rho_0^\circ$$

$$q_1^\circ = -{}^1/_2 w_1^{\circ\prime} - u_0^\circ, \quad \rho_0^\circ = \text{const}, \quad u_0^\circ = \text{const}$$

This system reduces to the following nonlinear differential equation for w_1°:

$$w_1^0(w_1^{\circ\prime\prime\prime} + 4w_1^{\circ\prime}) - u_0^\circ(w_1^{\circ\prime\prime} + 4w_1^\circ) = 0 \tag{4.6}$$

As can be verified directly, this equation has the following two-parameter family of periodic solutions:

$$w_1^\circ = A\sin 2\varphi + B\cos 2\varphi \tag{4.7}$$

where A and B are arbitrary constants.

Taking account of the no-leak condition at S ($w_0^\circ = 0$ for $\varphi = 0$ and $\varphi = \pi$) and also of the symmetry condition ($w_0^\circ = 0$ for $\varphi = \pi\ /\ 2$), we obtain the following formulas for the first coefficients of series (4.3):

$$w_1^\circ = A\sin 2\varphi, \qquad q_1^\circ = -A\cos 2\varphi - u_0^\circ \tag{4.8}$$

For the remaining coefficients we obtain a system of linear differential recursion equations with the appropriate boundary conditions.

Let us consider the limiting form of the isentropes. Their equation can be written as

$$\frac{d\theta}{\theta} \approx \frac{q_1^\circ d\varphi}{w_1^\circ}[1 + O(\theta)] \tag{4.9}$$

Substituting (4.8) and integrating, we obtain the equation of the isentropes

$$\theta \approx \frac{C(s)}{\sqrt{\sin 2\varphi}} \operatorname{ctg}^N(\varphi) \qquad \left(N = \frac{u_0^\circ}{2A}\right) \tag{4.10}$$

Alternatively, converting to the coordinates η, ζ by means of the formulas

$$\zeta \approx \theta \cos \varphi, \ \eta \approx \theta \sin \varphi + O(\theta^2)$$

and altering our symbol for the integration constant, which depends on the magnitude of the entropy function σ, we obtain

$$\eta = C_1(\sigma)\zeta^l \left(l = \frac{u_0^\circ - A}{u_0^\circ + A}\right) \tag{4.11}$$

It is clear that $q_1^\circ \neq 0$ for $|A| \neq |u_0^\circ|$, so that the stream surface S in this case correponds to an isentrope, and the neighborhood of the point O_2 is either a saddle ($|A| > |u_0^\circ|$) or a node ($|A| < |u_0^\circ|$) of the family of isentropes with an entropy singularity. In order to obtain the particular stream surface S_{O_2} we must ensure that the component q be equal to zero for $\varphi = 0$ and $\varphi = \pi$, i.e. we must set

$$A = -u_0^\circ \tag{4.12}$$

The subsequent coefficients of the expansion are obtainable from a system of ordinary differential recursion equations with boundary conditions corresponding to the no-leak condition at S_{O_2}.

Thus, the fact of existence of a symmetric solution (4.12) with a condition on S of the same type ($q = 0$, $w = 0$) as on the particular stream surface S_{O_2} indicates the possibility of two types of isentrope configurations in the vicinity of the point O_2 (Fig.6); moreover, the point O_2 is not an entropy singularity, since the isentropes do not converge to the point O_2, but terminate at S_{O_2} at disctinct points distributed continuously over S_{O_2}. We note that the case $A = u_0^\circ$ corresponds to S_{O_2} lying in the plane $\varphi = \pi / 2$.

5. Thus, in addition to regular stream surfaces coincident with isentropes, conical flows can also contain particular conical stream surfaces over which the entropy varies and which constitute envelopes of the family of isentropes. In this case the particular stream surfaces can merge continuously with the regular surfaces; they likewise admit of a symmetric solution without any entropy singularities either on the surface S or in the field of flow.

The above results suggest the existence of a new diagram of separation-free flow past conical bodies (Fig. 7b) different from the diagram proposed by Ferri [1] with its entropy singularity at the points of detachment of the streamlines (Fig. 7a). The surface of the conical body in the proposed diagram can be partially contiguous with the regular stream surface (e.g. the windward and side surfaces of a cone at angle of attack) and partially contiguous with the particular stream surface (the leeward portion of the cone in the neighborhood of the rear critical point). The entropy is constant on the first part of the

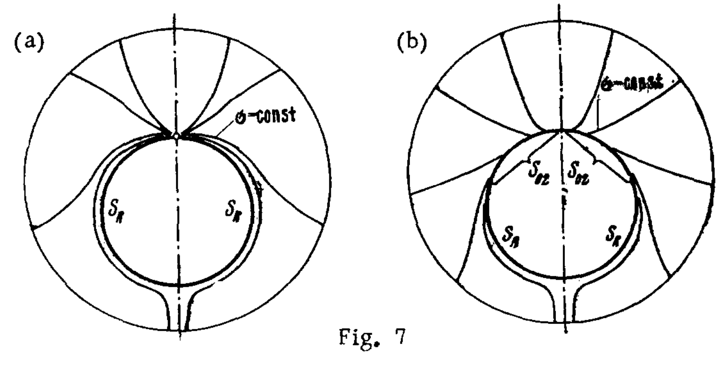

Fig. 7

surface (although the pressure and velocities vary); on the second part of the surface the pressure remains constant, while the entropy varies continuously from its value at the front part of the surface to its value at the rear critical point. It is apparently also possible to have still more complex flow diagrams containing several areas of constant and variable entropy.

The constant-entropy areas (i.e. areas of the type S_{O_1}) can be associated with fractional powers of the coordinates in the series and with more complicated patterns of joining of the respective areas. For this reason the flow diagram discussed above is not exhaustive and must be checked and refined by means of numerical calculations and experiments. We note that numerical calculations [8] have been carried out for relatively small angles of attack of a cone (smaller than the half vertex angle). This precludes their use for verifying the existence of the domain S_{O_2}, which must be very small in such cases.

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